

APPENDIX A

VOLUMETRIC BEHAVIOR AND STEADY STATE CONCEPTS

Part of: The Use of Waste Rock Inclusions to Control the Effects
of Liquefaction in Tailings Impoundments

A Ph.D. Thesis by Michael James
Ecole Polytechnique, Montreal

APPENDIX A

VOLUMETRIC BEHAVIOR AND STEADY STATE CONCEPTS

1. The Volumetric Behavior of Cohesionless Particulate Media

The volumetric behavior of a cohesionless soil (or cohesionless particulate media) subject to load is determined by the grain size distribution, the angularity of the soil grains, the confining stress, and the void ratio (or relative density). The particle structure may also affect the volumetric behavior, as will be discussed later. Figure B-1 presents the results of drained triaxial testing on samples of Sacramento River sand at various minor principle stresses. All of the samples were prepared at a relative density of approximately 38 percent, a “loose” condition. Part (a) of the figure presents axial strain, ϵ , versus principle stress ratio, σ_1'/σ_3' while part (b) presents axial strain versus volumetric strain, $\Delta V/V_o$. As shown on part (a) of the figure, all of the samples underwent positive axial strain (compression) during testing. In part (b) of the figure, it's shown that all of the samples contracted under the applied stresses except those tested at very low confining stresses (less than 200 kPa). Those tested at very low confining stresses underwent dilation albeit minor. Figure B-2 presents the results of similar testing on samples of the same sand compacted to relative densities of approximately 100 percent, a dense condition. Of the dense samples, those tested at confining stresses of less than 2000 kPa, dilated and those tested at confining stresses of 2000 kPa or greater contracted. Figures B-1 and B-2 illustrate the dependence of volumetric change on confining stress. Note that dense samples tested at higher confining stresses contracted in much the same manner as loose samples at higher confining stresses, although the loose samples underwent greater volumetric strains.

The primary mechanisms by which soils strain in response to applied stress is particle rearrangement by particles moving into tighter arrangements and thus causing the soil mass to contract or by particles sliding over and past one another into more spacious arrangements and thus causing the soil mass to dilate. The effect of confinement is that lower confining stresses favor particles sliding past one another in the direction of the confining stress causing dilation of the soil mass while higher confining stresses favor

particles seeking tighter arrangements or breaking and thus causing the soil mass to contract. Note that in both sets of tests (Figures B-1 and B-2) all of the axial strains are positive indicating compression in the direction of the major principle stress (Holtz & Kovacs 1981).

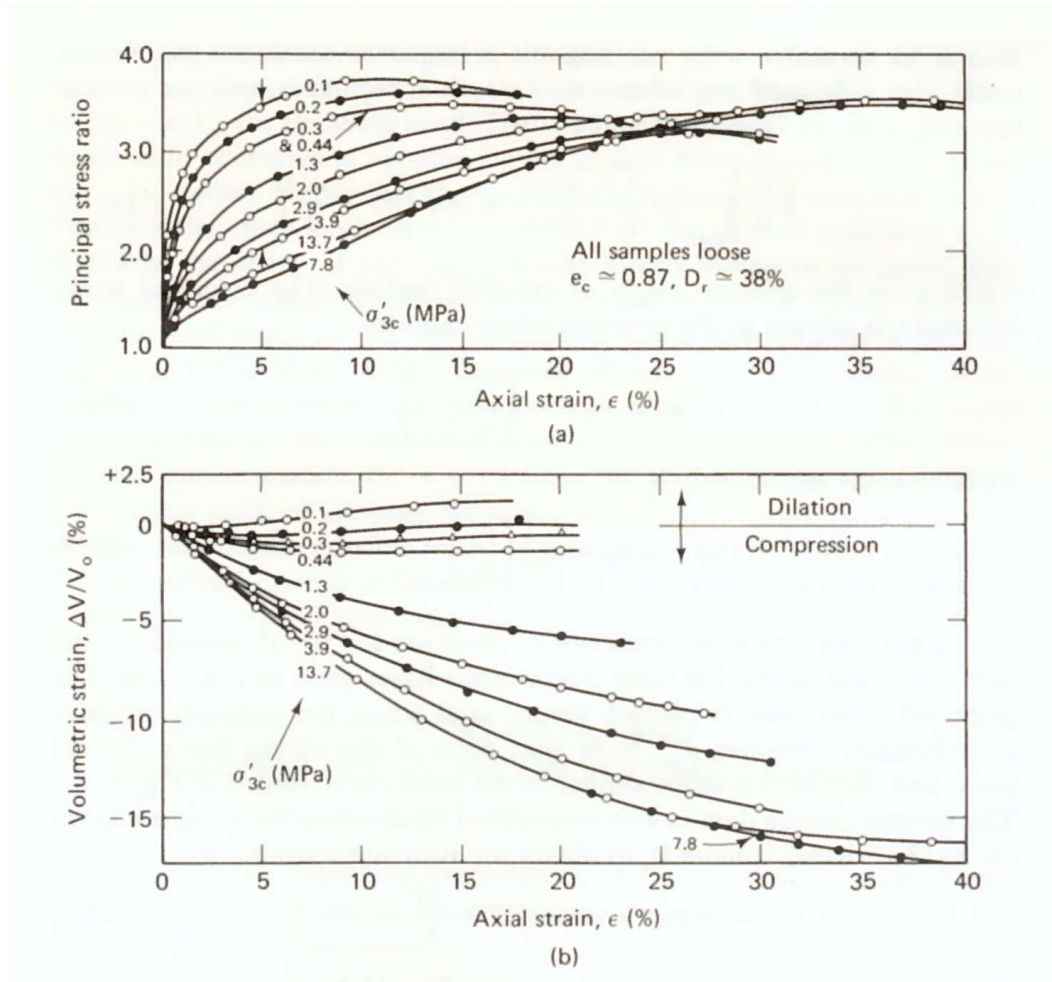


Figure A-1 – Typical Drained Triaxial Test Results on Loose Sacramento River Sand:
 (a) principle stress ratio v. axial strain; (b) volumetric strain v. axial strain
 (from Holtz & Kovacs 1981).

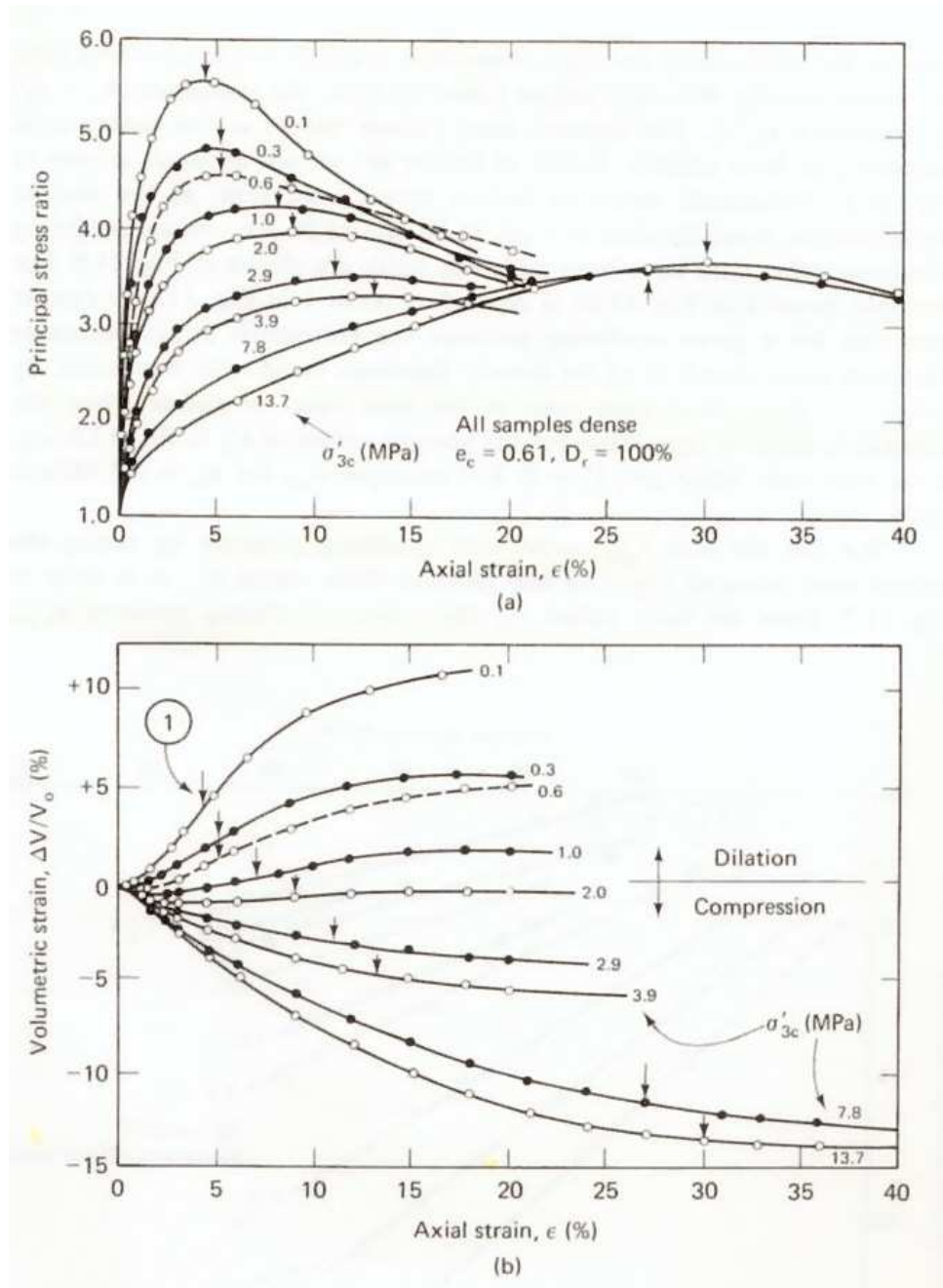


Figure A-2 – Typical Drained Triaxial Test Results on Dense Sacramento River Sand:
 (a) principle stress ratio v. axial strain; (b) volumetric strain v. axial strain
 (from Holtz & Kovacs 1981).

Figure B-3 presents plots of the void ratio versus volumetric strain at failure for the testing presented in Figures B-1 and B-2, where failure is the maximum principle stress

difference ($\sigma_1 - \sigma_3$) or the maximum stress ratio (σ_1'/σ_2'). All of the plots cross the abscissa, a condition of zero volumetric strain, $\Delta V/V_o$, neither contraction or dilation occurred when the soil at a given confining stress was at the corresponding void ratio. This void ratio is known as the critical void ratio, e_{crit} and is defined as the void ratio at which continuous deformation occurs with no change in stress (Holtz & Kovacs 1981; Been et al. 1991). This 'steady state' of deformation described by the critical void ratio is discussed in the following section of this appendix.

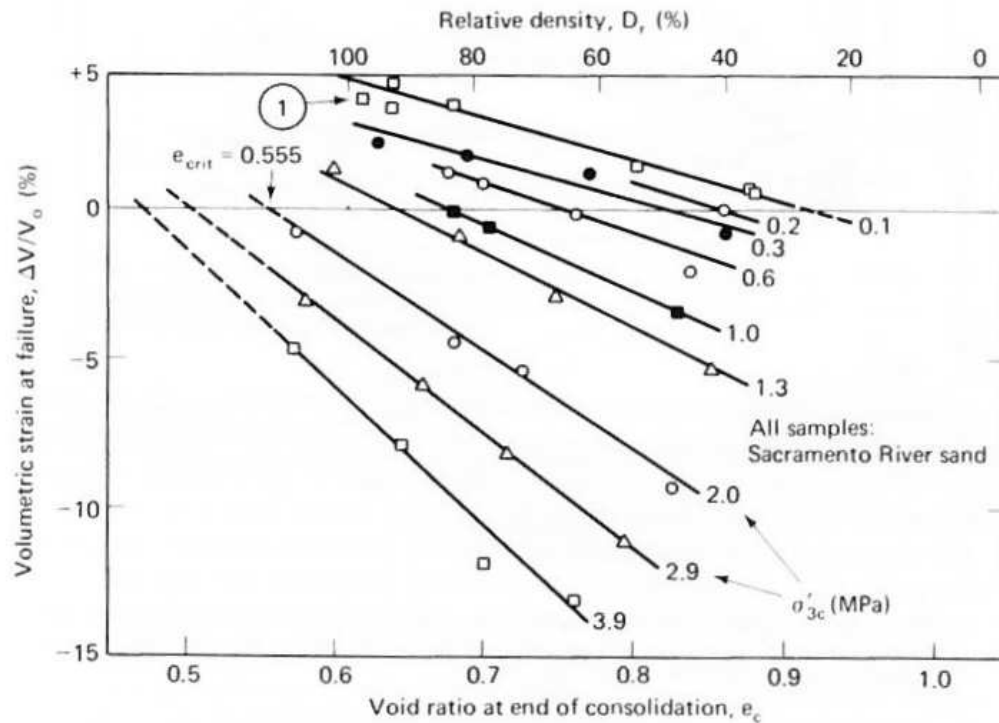


Figure A-3 – Volumetric Strain at Failure v. void ratio at end of consolidation for drained triaxial tests at various confining pressures (from Holtz & Kovacs 1981).

The critical void ratio represents a neutral state between contractive and dilative behavior for a soil at a given confining stress. This relationship can be determined for a given soil and plotted as shown on Figure B-4 for the referenced testing. The area above the curve in Figure B-4 indicates contractive behavior while the area below the curve indicates

dilative behavior. Similar curves can be developed for other soils and used to predict their volumetric behavior at a given confining stress and void ratio.

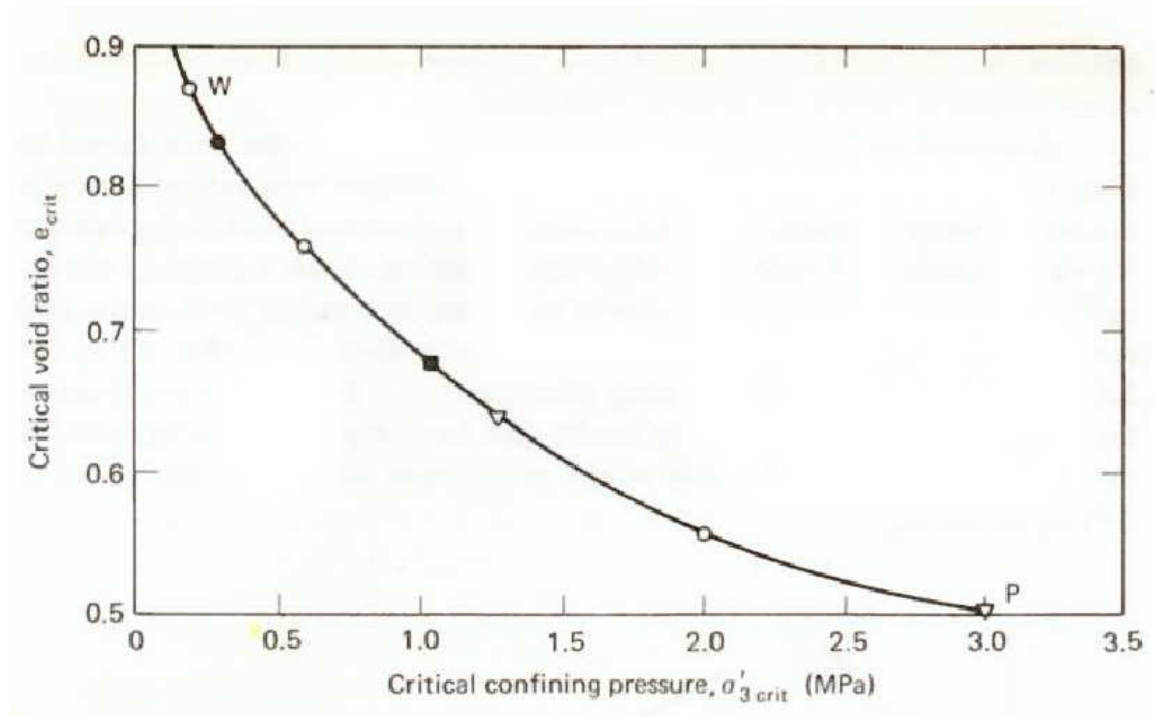


Figure A-4 – Critical Void Ratio v. Stress Conditions from Drained Triaxial Tests (from Holtz & Kovacs 1981).

Additional mechanisms can affect the volumetric behavior of cohesionless soil. These include the collapse of instable soil structures such as found in aeolian deposits of silt and sand and particle breakage at high confining stresses. Soil structure collapse can result in contraction where dilation is expected. The magnitude of confining stress required to make particle breakage a significant factor in volumetric behavior depends on the angularity and durability of the soil particles (Holtz & Kovacs 1981 and Been et al. 1991).

2. Steady State Theory

The critical state or steady state of a cohesionless soil mass is defined as a state where the soil mass deforms continuously at constant volume, constant stress conditions, and constant velocity. The state is achieved when the soil structure is in a condition where it can continuously deform under the existing stress without additional particle breakage. (Holtz & Kovacs 1981; Been et al. 1991). Figure B-4 (page B-7) presents the results on drained triaxial testing on a loose and dense samples of a typical sand which show the tendencies for both samples to approach the steady state condition, the critical void ratio. The fact that neither sample actually achieves a true steady state condition is theorized to be due to inaccuracies in the testing and measurement equipment and non-uniform sample deformation, e.g. necking of the sample (Been et al. 1991).

Figure B-5 is an example of a steady state diagram developed from a series of undrained triaxial tests on contractive samples of sand by Been et al. (1991). The sands used in the testing was uniformly graded, fine to medium grained, with subrounded particles.

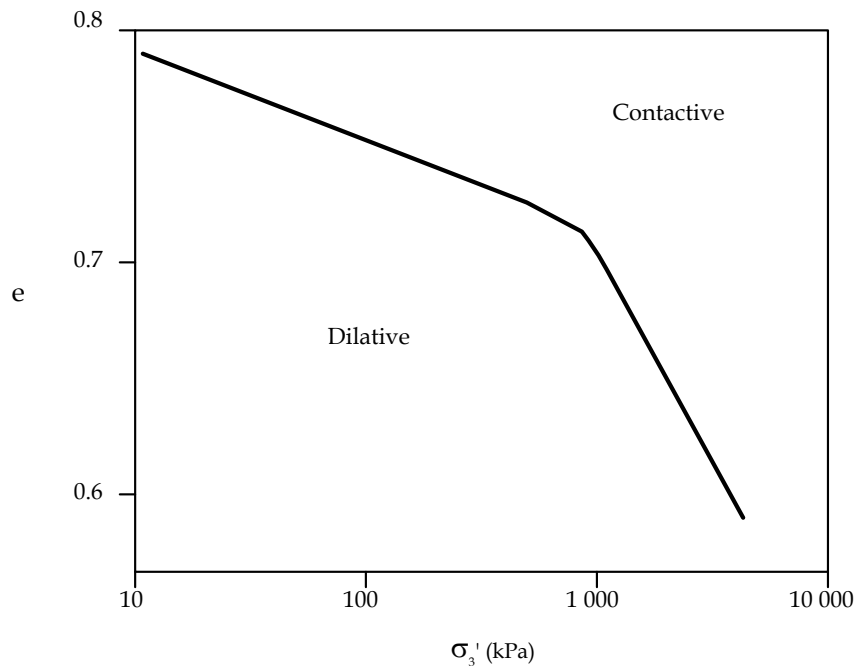


Figure A-5 – Example Steady State Diagram (after Been et al. 1991).

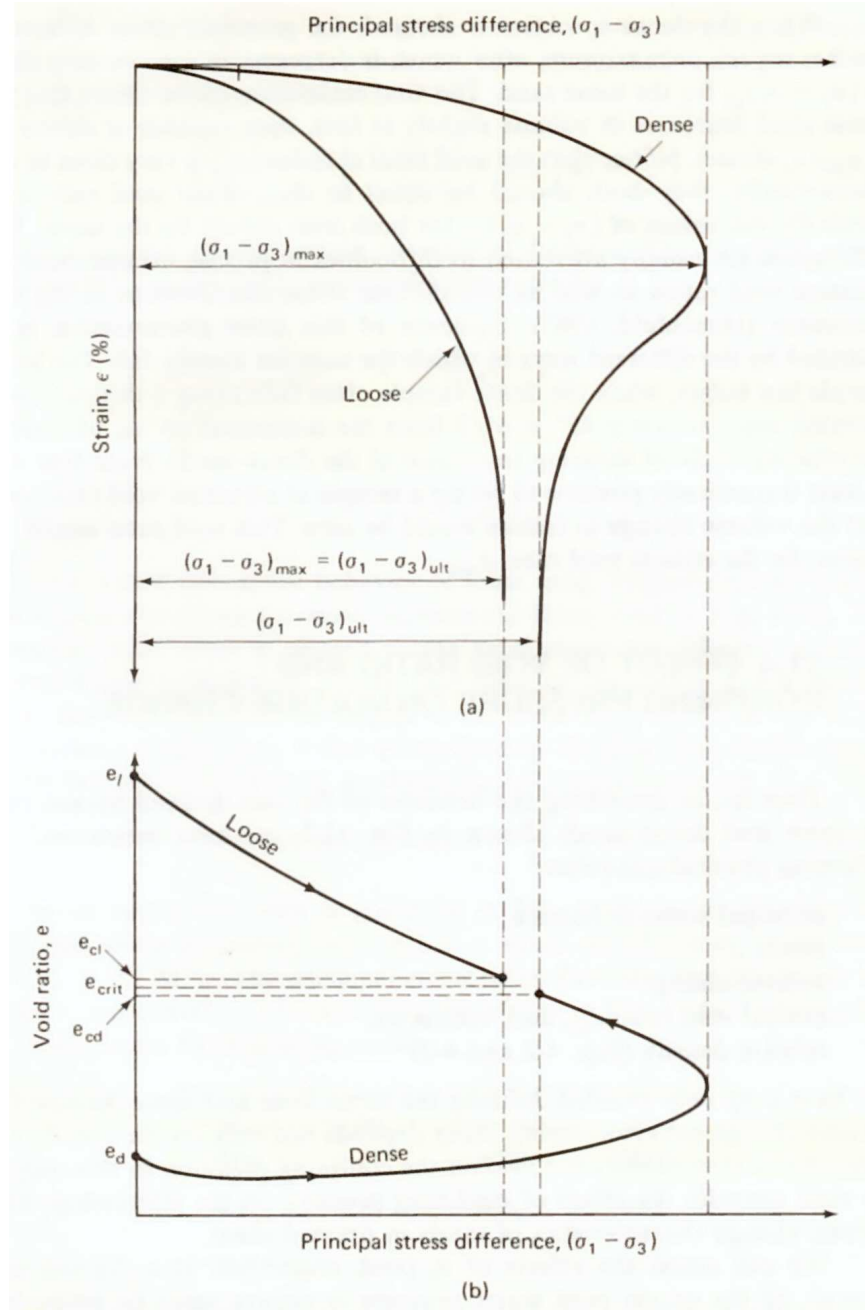


Figure B-4 – Typical Triaxial Test Results on Loose and Dense Sand: (a) Stress-strain Curves; and (b) stress void ratio curves.
(from Holtz & Kovacs 1981).

Triaxial testing can be used to create a similar curve for any non-cohesive soil. The primary factors which govern the volumetric behavior of the soil and thus the shape of the curves are the grain size distribution and the angularity of the particles. The steep slope of the curve beyond a confining stress of 1000 kPa is due to the effect of particle breakage at higher confining stresses. The location of this change in slope is expected to vary with grain size distribution, particle shape and particle hardness and it may not occur in the case of uniformly graded, round, hard particles.

The steady state line for sands can be approximated by a straight line on a graph of void ratio versus confining stress where the confining stress is less than 500 kPa. (Been et al. 1991).

Since liquefaction of a soil mass is predicated upon the tendency of a soil to contract under applied load, steady state diagrams can be used to evaluate whether a particular soil may be susceptible to liquefaction as opposed to cyclic mobility. As discussed in the report, methods for evaluating the liquefaction potential of soil based on steady state diagrams have been developed and are currently in use.

Vaid & Thomas (1995) demonstrated that the volumetric response of sand under extensile loading differs significantly from that under compressive loading. Therefore, this discussion has been limited to the volumetric response of soil under compressive loads only.